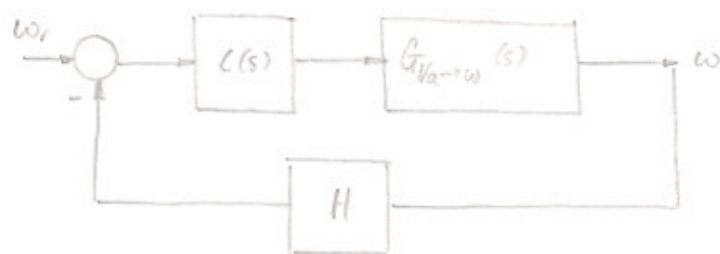


$$G_{v_a \rightarrow \omega}(s) = \frac{k_e I_f}{(Js + D)(La + Ra) + (k_e I_f)^2}$$

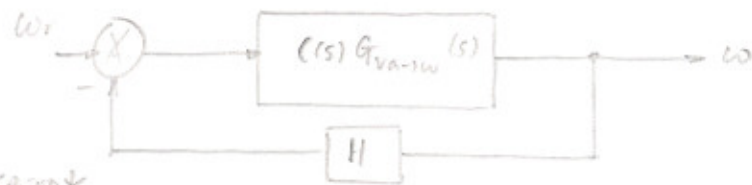
$$C(s) = k_p + \frac{k_I}{s} + k_D s = \frac{k_D s^2 + k_p s + k_I}{s}$$

→ Let $V_b = 0, T_L(s) = 0$ for controller design.

* During testing, if $V_b = 0$, the zero voltage resulting from $\omega_r = \omega$ will force the controller to apply 0V to the motor armature \Rightarrow motor speed will drop. Applying $V_{bias} = V_{arm, full speed}$ will ensure that when error = 0, motor runs @ full speed *



* If motor is too slow we need a +ve error, which will be "processed" to increase motor speed *



$$G_c(s) = \frac{C(s) G_{v_a \rightarrow \omega}(s)}{1 + H C(s) G_{v_a \rightarrow \omega}(s)}$$

$$= \frac{\left(\frac{k_D s^2 + k_p s + k_I}{s} \right) \left(\frac{k_e I_f}{(Js + D)(La + Ra) + (k_e I_f)^2} \right)}{1 + H \left(\frac{k_D s^2 + k_p s + k_I}{s} \right) \left(\frac{k_e I_f}{(Js + D)(La + Ra) + (k_e I_f)^2} \right)}$$

→ $G_{eqn}(s) =$
closed loop TF.

$$\frac{(k_e I_f)(k_D s^2 + k_p s + k_I)}{s(Js + D)(La + Ra) + (k_e I_f)^2 + H k_e I_f (k_D s^2 + k_p s + k_I)}$$

energy

$$F = - \frac{\partial W(i, x)}{\partial x}$$

$$= \frac{\partial W(i, x)}{\partial x}$$
 co-energy

For controller design we need only consider the denominator of $G_c(s)$

$$G_c(s) = \frac{k_e I_f}{J \cdot L a} (k_D s^2 + k_P s + k_I)$$

$$\underbrace{s^3 + \left(\frac{J R a + D L a}{J L a} \right) s^2 + \frac{D R a}{J L a} s + \frac{(k_e I_f)^2}{J L a}}_{1^{st}} + \underbrace{\frac{H k_e I_f k_D s^2 + H k_e I_f k_P s + H k_e I_f k_I}{J L a}}_{2^{nd}}$$

$$G_c(s) = \frac{k_e I_f}{J \cdot L a} (k_D s^2 + k_P s + k_I)$$

$$s^3 + \left(\frac{J R a + D L a + H k_e I_f k_D}{J L a} \right) s^2 + \left(\frac{D R a + (k_e I_f)^2 + H k_e I_f k_P}{J L a} \right) s + H k_e I_f k_I$$

CONTROLLER:

1ST ORDER:

$$C(s) = \frac{K}{Ts + 1}$$

* 2ND ORDER: *

$$C(s) = \frac{K \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$

3RD ORDER:

$$\frac{N(s)}{(s^2 + 2\zeta \omega_n s + \omega_n^2)(s + p_3)}$$

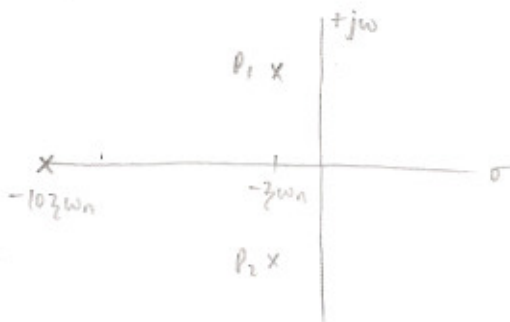
$$t_s = \frac{3}{\zeta \omega_n}, \quad 0.5 = e^{\frac{-\zeta \pi}{\sqrt{1-\zeta^2}}}$$

$$p_{1,2} = -\zeta \omega_n \pm j \omega_n \sqrt{1-\zeta^2} \quad (\zeta \leq 1)$$

p_3 is placed further away from origin, make it a real pole, $p_3 = -10\zeta \omega_n$

* $p_{1,2}$ should be dominant. *

$p_3(t) \rightarrow$ should be quick to "fade out" \rightarrow large $-ve$ pole $\rightarrow e^{-\dots}$: quick to end.
 $e^{Re(p_1)t}$
 $e^{Re(p_2)t}$ } DOMINANT



* $G_{\text{equiv}}(s)$ is made to match $G_c(s)$ so that we can solve for k_I, k_p, k_d .

$$G_{\text{equiv}}(s) = \frac{N(s)}{s^3 + (P_3 + 2\zeta\omega_n)s^2 + (2\zeta\omega_n P_3 + \omega_n^2)s + P_3\omega_n^2}$$

Match the two denominators and equate coefficients:

$$P_3 + 2\zeta\omega_n = \frac{JRa + DLa + HkeI_f k_d}{JLa}$$

$$2\zeta\omega_n P_3 + \omega_n^2 = \frac{DL_a + (keI_f)^2 + HkeI_f k_p}{JLa}$$

$$P_3\omega_n^2 = HkeI_f k_I$$

∴ First we must select settling time, t_s , and overshoot, o.s.:

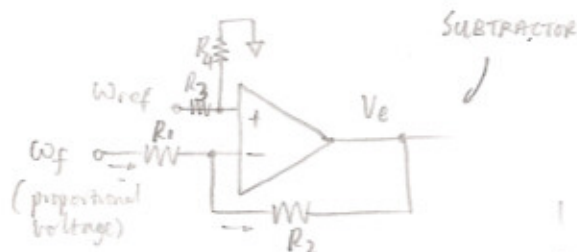
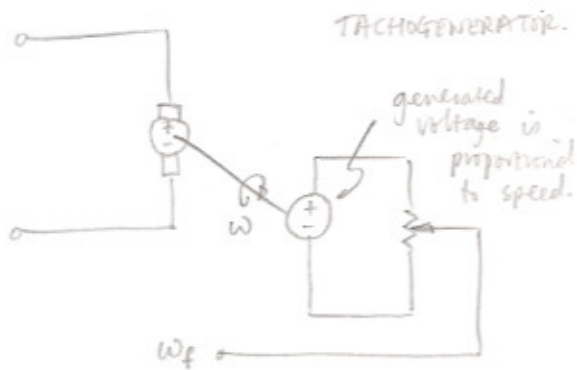
$$t_s = \frac{3}{\zeta\omega_n}, \quad \text{o.s.} = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

These equations give ω_n and ζ .

$$\text{eg. } k_I = \frac{P_3\omega_n^2}{HkeI_f} \dots$$



Process requires knowledge of motor model, including J and D .
Tune K error required if the model is not known.

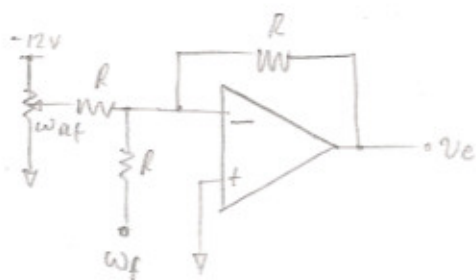


$$V^+ = \frac{\omega_r \cdot R_4}{R_3 + R_4}$$

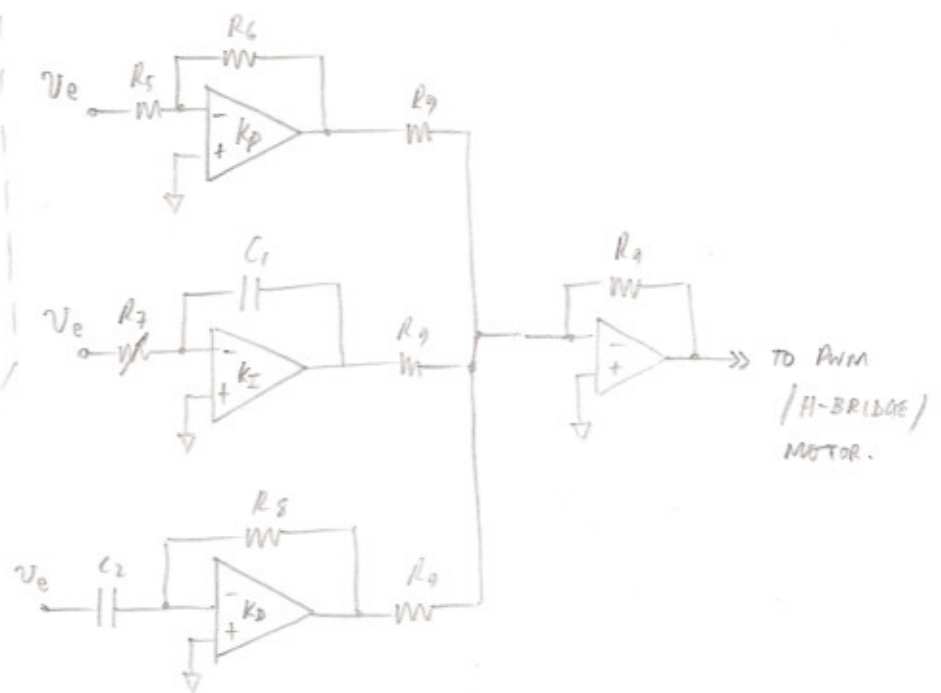
$$\frac{\omega_f - V^+}{R_1} = \frac{V^+ - V_e}{R_2}$$

$$V_e = \omega_r - \omega_f$$

OR: SUMMING AMP.



CONTROLLER:



MC PROPORTIONAL CONTROLLER

